

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

TMA 1301 – COMPUTATIONAL METHODS

(All sections / Groups)

7 MARCH 2016
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **8 pages** only with **4 Questions** and an **Appendix**.
2. Attempt ALL **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each solution of a question on a new page**.
4. Show all steps.

Question 1

a) Given the polynomial function,

$$g(x) = x^5 + x^4 - 2.1x^3 - 1.4x^2 + 3.8x + 4.9$$

(i) Turn the given function into *nested form*.

[2 marks]

(ii) Calculate the value of $f(0.889)$ using (i) and **five-digit arithmetic** with rounding.

[2.5 marks]

(iii) Taking 6.8761 as an actual value of $f(0.889)$, find the **absolute error** for the computed value from (ii).

[0.5 mark]

b) Good random numbers play an important role in Monte Carlo simulations. State **two** properties of a good random number generator.

[2 marks]

c) Given the multiplier $a = 7$, increment $c = 4$, modulus $m = 9$ and the initial value $x_0 = 5$, apply Mixed Linear Congruential Generator (LCG) to generate a series of 6 random numbers.

[3 marks]

Continued.....

Question 2

[Note: For this question, use **four decimal places** for all the workings.]

- a) Suppose $f(x)$ is a function on interval $[0, \frac{\pi}{2}]$ with

$$f(x) = x - \cos x$$

- (i) Perform **three** iterations using **Newton's method** to reach an approximated root of the equation $f(x) = 0$, starting with initial value $p_0 = 0$. **Show** the working steps.

[4 marks]

- (ii) If the actual root of the equation $f(x) = 0$ is 0.73909, compute the **absolute error** of the approximated root.

[0.5 marks]

- b) (i) Using the **Trapezoidal rule** with step size $h = 0.25$,

approximate $\int_0^1 \frac{1}{9 + 4x^2} dx$.

[3 marks]

- (ii) Evaluate the definite integral for $\int_0^1 \frac{1}{9 + 4x^2} dx$

[2 marks]

- (iii) Hence, compute the **absolute error** for the approximation from (i) (Take the approximation of the answer to (ii) correct to four decimal places as the actual value).

[Note: $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$]

[0.5 mark]

Continued.....

Question 3

a) Use row reduction technique to find an upper triangular, U , in the LU factorization of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

[4 marks]

b) Compute the first iteration of the *Jacobi Method* for the following linear system.

$$7w - 2x + y + 2z = 3$$

$$2w + 8x + 3y + z = -2$$

$$-w + 5y + 2z = 5$$

$$2x - y + 4z = 4$$

Copy the following table into your *Answer Booklet* and *complete* it. Write your answers correct to six decimal places.

n	w	x	y	z
0	0	-1	1	1
1				

[4 marks]

c) Find the *eigenvalues* for the following matrix A :

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

[2 marks]

Continued.....

Question 4

a) The data in the following table relates the age of CDROM in months to the percentage of damaged surface areas.

Age (X)	9	12	12	15	18	18	21	21	27	30	30	33
Percentage of damaged (Y)	11	12	23	30	29	52	41	65	60	72	84	93

(i) *Copy* the following table into your *Answer Booklet* and *complete* it.

x	y	x^2	xy
9	11		
12	12		
12	23		
15	30		
18	29		
18	52		
21	41		
21	65		
27	60		
30	72		
30	84		
33	93		
$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum xy =$

[2 marks]

[Note: round the answer in part (ii) and (iii) to 5 decimal places]

(ii) From (i), find the *equation of the best fit linear line* using the *method of least squares*.

[2.5 marks]

(iii) *Estimate* the value of y when $x = 25$.

[0.5 marks]

Continued.....

b) Given the following data:

x	0	2	4	6
$f(x)$	1	2	4	8

(i) Construct a divided difference (DD) table.

[2 marks]

(ii) Find the third-degree Newton Polynomial $P_3(x)$ using the result from (i).

[2 marks]

(iii) Hence, approximate $f(3)$. Write your answer correct to five decimal places.

[1 mark]

End of Exam Questions.

APPENDIX - FORMULAS

1. LOCATING ROOTS OF EQUATIONS

(A) Bisection Method

$$p_n = \frac{b_n + a_n}{2}$$

(B) Secant Method

$$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$$

(C) Newton's Method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

2. NUMERICAL INTEGRATION

(A) Trapezoidal Rule

$$A_T = \int_a^b f(x)dx \approx \frac{h}{2} \{f(x_1) + f(x_{n+1}) + 2[f(x_2) + f(x_3) + \dots + f(x_n)]\}.$$

(B) Simpson's Rule

$$A_S = \int_a^b f(x)dx \approx \frac{h}{3} \left\{ f(x_1) + f(x_{2n+1}) + 4[f(x_2) + f(x_4) + \dots + f(x_{2n})] \right. \\ \left. + 2[f(x_3) + f(x_5) + \dots + f(x_{2n-1})] \right\}$$

Continued...

2. NUMERICAL INTEGRATION (CONTINUED)

(C) Romberg Algorithm

$$R(0,0) = \frac{1}{2}(b-a)[f(a) + f(b)]$$

$$R(n,0) = \frac{1}{2}R(n-1,0) + h \sum_{k=1}^{2^{n-1}} f[a + (2k-1)h]; \quad h = \frac{(b-a)}{2^n}, \quad n \geq 1$$

$$R(n,m) = R(n,m-1) + \frac{1}{4^m - 1} [R(n,m-1) - R(n-1,m-1)],$$

where $n \geq 1, m \geq 1$

3. LEAST SQUARES PROBLEMS, INTERPOLATION AND POLYNOMIAL APPROXIMATION

(A) Linear Least Squares $y = a + bx$

$$a = \frac{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i - \sum_{i=1}^n x_i y_i \sum_{i=1}^n x_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}, \quad b = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

(B) Taylor Polynomial

$$P_n(x) = \sum_{k=0}^n \frac{1}{k!} f^{(k)}(a)(x-a)^k$$

(C) Lagrange Polynomial

$$P_n(x) = \sum_{i=0}^n f(x_i) L_i(x)$$

End of Page